A Study of Streams of Solids Flowing From Solid-Gas Fluidized Beds

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The wide variety of the applications of the fluidized-bed techniques to processing operations in the chemical and allied industries can be explained in large part by the fact that the fluidized suspensions consisting of solids and gases have some physical characteristics similar to those of bodies in the solid state (such as density, thermal conductivity, and specific heat), and yet at the same time these suspensions have the mobility of fluids. Further, it is possible to combine suspensions of the same materials which have different characteristics because of density, velocity, and direction in the same equipment. This combination gives a flexibility and control of operation which could not be readily achieved by other process techniques.

Particularly useful in the application of fluidized systems is the ease with which streams of the solid-gas mixture can be withdrawn or added to the processes. These solid-gas streams can be removed from the system by allowing them to flow from a region of high pressure to an exterior region at a lower pressure. If an orifice is set into the side of the tube containing the fluidized bed, a flow of a solid-gas mixture may occur because of the pressure difference between the bed and the atmosphere. The flow rate through an orifice of fixed size can be controlled by varying the height of the fluidized bed; the greater the bed height the greater the fluid pressure at the bottom necessary to cause fluidization and thus the greater the pressure difference across the orifice.

In this study the efflux of solid and gas from fluidized beds has been investigated by using the experimental arrangement shown in Figure 1. With this arrangement the gas moves upward, the solid downward, and the inlet flow rate of the solid is adjusted to correspond to the rate of efflux of the solid from the orifice. As a whole the system considered is a countercurrent bottom restrained fluid bed as defined by Lapidus and Elgin (3), but two distinct phenomena take place in this system: a flow of solid and gas in the column and a flow of solid and gas through the orifice. No attention is given in the study to the first of these phenomena, which has been extensively studied in previous works (4, 8) under a variety of experimental conditions. The purpose of the study was limited to the investigation of the characteristics of the solid-gas mixtures as they flow from fluidized beds and the variables which influence their rates of discharge. In order to do this the flow of solid and gas through orifices cut in the wall of the unit was measured. Also the ratio of solid to gas was determined as a function of the characteristics of the fluidized system, such as the height of the bed, the pressure drop across the orifice, and the mass velocity of the fluidizing gas. Solids having different properties and dimensions were employed. Their characteristic properties are found in Table 1. Air was used as the fluidizing gas in all experiments.

This work is divided into two major parts. In the first there is a description of the apparatus, the experimental techniques used, and the data obtained. In the second part equations are developed on the basis of theoretical considerations to give the flow rate of a solid-gas mixture through an orifice, and a comparison is made between the actual results and those predicted by the equations. Reasons for the discrepancies which occur are advanced.

EXPERIMENTAL APPARATUS

The general layout of the equipment is shown in Figure 1. This equipment con-

TABLE 1. CHARACTERISTICS OF THE SOLIDS USED

Solid	Density, kg./cu. m.	Size distribution, mm. from to	%	Average diameter, mm.	Shape factor in Carman-Kozeny correlation
Silica sand	2,600	0.35-0.29	8.8	0.22	0.80
		0.29-0.24	36.8		
		0.24-0.17	28.4		
		0.17-0.13	23.0		
		0.13-0.12	3.0		
Sawdust	700	0.35-0.29	38.0	0.27	0.50
		0.29-0.24	43.0		
		0.24-0.17	14.0		
		0.17-0.13	5.0		
Coke	1,500	0.55-0.29	50.0	0.33	0.70
		0.29-0.24	34.0		
		0.24-0.17	13.1		
		0.17-0.13	2.9		
Limestone	2,700	0.55-0.29	46.0	0.32	0.80
		0.29-0.24	31.1		
		0.24-0.17	17.8		
		0.17-0.13	5.1		
Glass beads A	2,600	0.74-0.55	31.2	0.47	0.90
		0.55-0.24	67.0		
		0.24-0.17	1.8		
Glass beads B	2,600	1.00-0.70	76.5	0.80	0.90
		0.70-0.55	22.0		
		0.55-0.35	1.5		
Magnetite	5,200	0.35-0.29	16.3	0.26	0.80
		0.29-0.24	57.3		
		0.24-0.17	21.4		
		0.17-0.13	5.0		

sists essentially of a column which contains the fluidized bed, a source of air for fluidizing the bed, an arrangement for adding solids to the fluidized bed, an arrangement for removing a mixture of solids and air simultaneously through an orifice, an apparatus for measuring the relative amounts of solid and gas which have passed through the orifice, and finally a manometer for measuring the pressure at the base of the fluidized bed.

The column (l) in which the fluidization occurred was made of a tube of perspex 90-mm. I.D. and 1,200 mm. high. At the bottom there was a flange which was used to connect the column to another tube of the same diameter. The fluidizing gas entered this lower tube and came from a 5-hp. blower (a). At the bottom of the column was placed a porous glass plate (g), which distributed the air flow. Circular screens (m) were inserted in the column from top to bottom. These screens were 13 mm. apart and were made of galvanized iron wire 1 mm. in diameter. The mesh was square and measured 5 mm. on edge. The purpose of these wire screens was to make the fluidization uniform.

The pressure drop across the column was measured by use of a differential water manometer (h). The upper pressure tap was at the top of the column and the lower one 5 cm. above the porous plate and at the same level as the orifice for the solid-gas jet. In order to minimize the oscillations of the liquid in the manometer which resulted from fluctuations in the pressure in the column a 10-liter ballast tank was inserted in each of the pressure lines between the column and the manometer. Five centimeters from the bottom of the column the efflux orifices were located. The diameters of the orifices used were 3, 3.5, 4, 5.25, and 7 mm. The detail of the orifice design is shown in Figure 1 (n). These orifices were cut into the column and a larger tube, 40 mm. in diameter and 20 mm. long, was cemented to the column at each orifice and had the orifice as its center.

The air for the fluidization was pumped by the blower to the base of the column. A valve (b) regulated the quantity of air flowing to the column, and in the pipe between the blower and the base of the column was inserted an orifice (f) for measuring the quantity of air entering the column.

The solids to be fed to the column were contained in a storage funnel (i) and passed through two valves. The upper valve (j) was used as an on-off valve, and the lower valve (k) was used to regulate the flow of solids to the column.

The solids which flowed from an orifice at the base of the column were collected in an air-tight container made of perspex (o) which was 18 liters in size. This container has four openings in it. One is used to connect the collector to the column. Opposite this connection was a hole containing a rubber stopper. Through this rubber stopper was a long thin steel rod (s) which could be used to make sure that the suspension flowed freely into the container. At the bottom the container terminated in a tube (p) which could be quickly closed when one began to collect the solid at the beginning of a test. At the top of the container was an exit through which the gas which collected in the container could flow to the device to measure it.

The air was collected by displacement of water in a 28-liters container (v) which terminated at the bottom in a graduated cylinder. This cylinder was used to reduce the error in the measurement of the final level of the water. The air was collected at atmospheric pressure in all of the experiments by the following procedure. A constant vacuum was maintained at the point where the air entered the container by having the entrance placed 50 cm. above the water level. This level was maintained constant by an overflow device (x). This vacuum is more than sufficient to overcome the pressure drop which occurs in the passage of the air through the tube. The excess suction is balanced by inserting into the tube connecting (o) and (v) an adjustable valve (u). A water manometer (r) was also connected to the tube in order to know the pressure during the entire time of the run. In the tube was also inserted a three-way valve (t) which permitted the rapid inclusion of the measuring container (v) into the system. The container is refilled with water by removing the collected air by means of a vacuum pump connected to the tube (w).

The flow rates of air and of solid were determined by measuring the volume of solid and air collected and dividing by the time of collection. The value of the flow rate of the air was corrected taking into account the volume of air displaced by the solid in the vessel in which the solid was collected. The measurements of the flow of gas and solid were done twice for the same conditions of efflux. The results were reproducible within 3%.

EXPERIMENTAL PROCEDURE

The experimental procedure employed was essentially the same for all runs except when sawdust was used. In each run a sufficient quantity of solid was put into the column to give a desired bed height; the air was turned on and the bed gradually fluidized. Measurements were made of the pressure drop across the bed at various air rates and various operating bed heights. These measurements were made both with an increasing and decreasing flow rate in order to check the work. During this period the orifice was plugged. This plug was then removed and a connection made to the collector (o); then the flow of solids to the fluidized bed from (i) was regulated in such a fashion that it equaled the quantity flowing through (n) into (o) and out at (p). When the flow rates of the solid to and from the system were constant and when the bed height was at the desired level, the opening (p) was closed, (t) was opened, and by means of the pinch clamp (u) atmospheric pressure was maintained in vessel (o). The time of collection of the solid in (o) and gas in (v) was recorded.

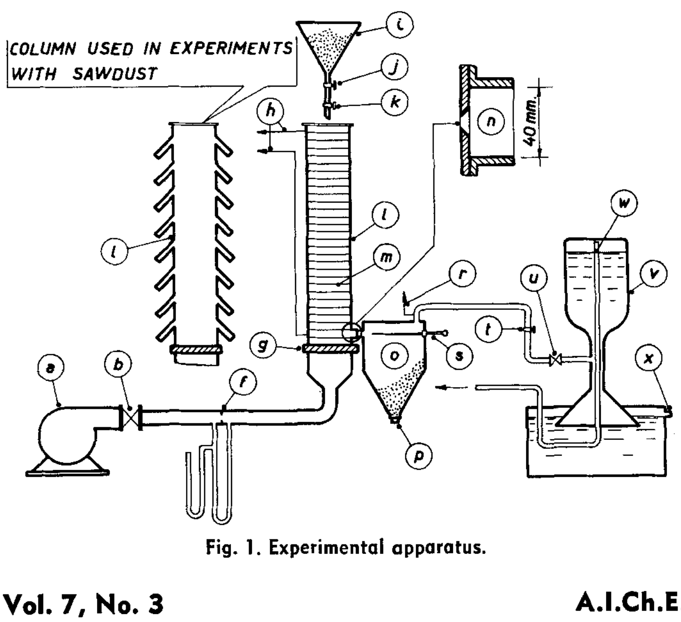

Fig. 1. Experimental apparatus.

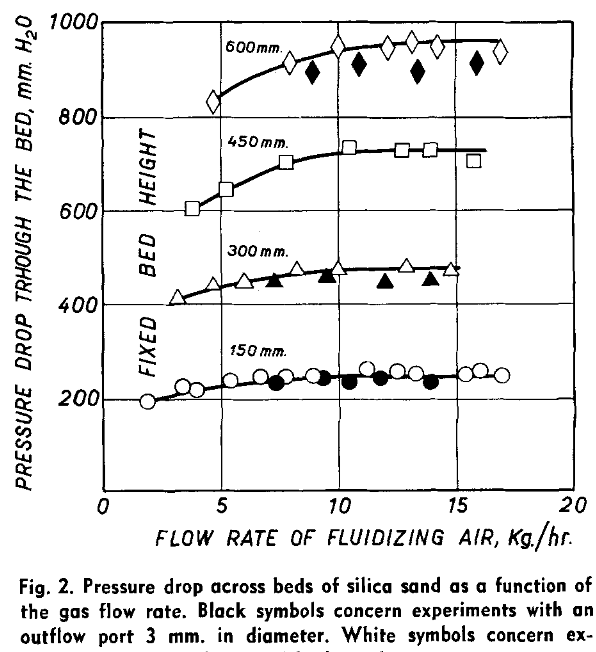

Fig. 2. Pressure drop across beds of silica sand as a function of the gas flow rate. Black symbols concern experiments with an outflow port 3 mm. in diameter. White symbols concern experiments with plugged ports.

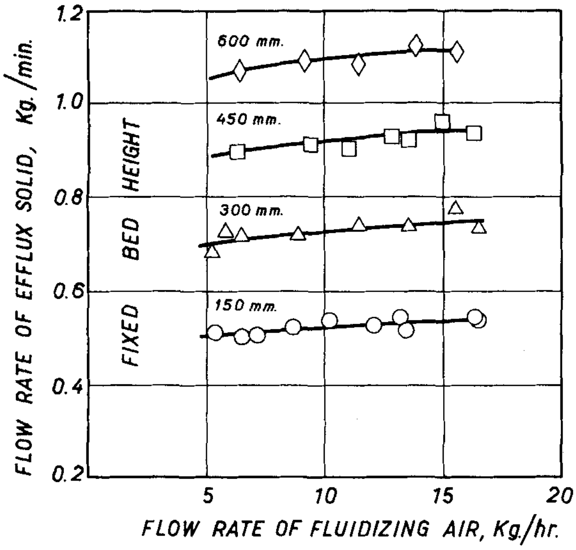

Fig. 3. Experiments with silica sand. Flow rate of solid from the orifice as a function of the gas flow rate. Port diameter = 3 mm.

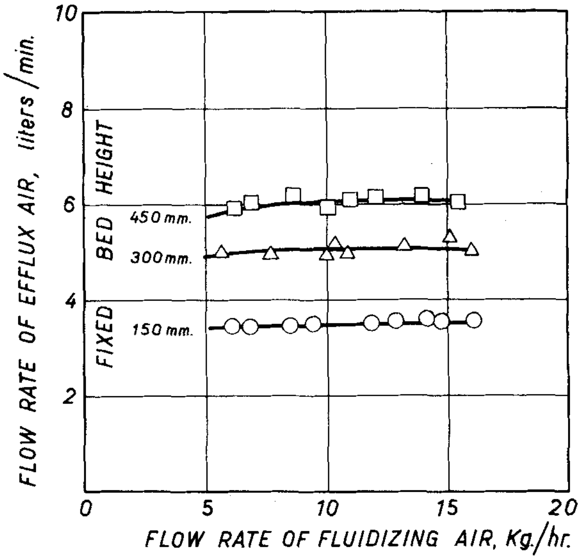

Fig. 4. Experiments with silica sand. Flow rate of air from the orifice as a function of the gas flow rate. Port diameter = 3 mm. The air flow rate has been measured at atmospheric pressure and at 20°C.

The variables and their ranges were as follows:

Air flow rate	2 to 18 kg./hr.
Fixed bed height	150, 300, 450, and 600 mm.
Orifice diameter	3, 3.5, 4, 5.25, and 7 mm.
Materials	Silica sand, magnetite, limestone, coke, glass beads, and sawdust.

Special procedure with sawdust

When sawdust was used as the solid in the fluidized bed, a different procedure had to be devised to feed the sawdust to the fluidized bed because the funnel apparatus did not work. A different kind of column was constructed, as shown in Figure 1, which permitted an excess of sawdust to be fed to the fluidized bed, and any excess in addition to that which did not flow out of the bottom through the discharge orifice flowed through ducts placed in the side of the column. The discharge tubes were 3 cm. in diameter, and two were placed at each level in a position diametrically opposite to each other. The lowest discharge tubes were 15 cm. from the bottom, and successive ones were each 10 cm. apart.

In making a run the height of the fluidized bed was decided upon, and all of the discharge tubes below this height were closed. Then the sawdust was poured into the column at such a rate that a fluidized bed of the desired height existed when the air flow was at a fixed value. The efflux orifice was then opened and a run started. In almost all of the tests the air flow was 8 kg./hr. By using the proper discharge tubes the height of the sawdust could be fixed and thus the pressure head on the efflux orifice established.

EXPERIMENTAL RESULTS

Some typical experimental data are shown in Figures 2 through 4 for one set of experiments with the silica sand. The detailed experimental results are found in references 1 and 2.

The results shown in Figure 2 demonstrate that the pressure drop across the column varies little with different flow rates of the fluidizing air. The maximum percentage difference between the pressure drop existing at the flow rate at which fluidization begins and the pressure drop at the maximum air flow rate was only 15 to 20%. These pressure drop, gas flow diagrams are similar to those obtained with unbaffled fluidized beds, even though screens were present in these columns.

A comparison, shown in Figure 2, between the pressure drop obtained when the efflux orifice was closed and when it was open showed that the effect of the lateral outflow on the pressure drop was negligible below bed heights of 450 mm. and it did not amount to more than a 10% change at higher bed heights.

The data of Figures 3 and 4 show that the quantity of solid and gas passing out of the efflux orifice is only slightly influenced by the quantity of the fluidizing gas and also by the corresponding value of the degree of voids, which, as is well known, depends directly upon the flow rate of fluidizing gas. The small differences which were observed are attributed to the fact that as the flow rate of the air increases, the pressure drop across the column increases slightly (Figure 2). These observations simplified the experimental work, since they showed that it was not necessary to vary the air flow rate for every bed height.

A preliminary analysis of the results revealed that the volumetric ratio of the gas to solid flowing out of the orifice was completely different from the volumetric ratio of gas to solid in the fluidized bed.

The degree of voids for the fluidized beds considered in this study are of the order of 0.45 to 0.70, while the degree of voids $Q_A/(Q_A + Q_S)$ for the effluent ranges from 0.90 to 0.98.

Therefore this phenomenon cannot be considered simply as an outflow of the solid-gas suspension which exists in the fluidized bed and so it cannot be studied with the usual formulas valid for the flow of a continuous medium:

$$\Delta p = \frac{\xi \, \rho_F \, V_F^2}{2 \, g_c} \tag{1}$$

with ξ slightly different from 1. These formulas are derived by equating the potential to the kinetic energy of the effluent, respectively, on the upstream and downstream side of the orifice. Thus their application would presuppose the effluent to be a fluid having the same characteristics as the fluid in the column, that is the same degree of voids.

Everything shows instead that immediately upstream of the outflow port separation occurs of the solid from the solid-gas suspension present in the column. In fact it is necessary to take account of the fact that the system which flows out is not homogeneous but is composed of two phases, one solid the other gas, each of which because of different physical characteristics plays a different role with respect to flow from the orifice. The pressure drop from the upstream to the downstream side of the orifice is transmitted through the air and produces the flow of air. The flowing air drags the solid through the orifice, and at the same time the air is filtered by the solid.

It also appears that the energy which the flowing air loses by friction in filtering through the particles is very large. In most of the experiments carried out in this study the amount of kinetic energy of the air and solid on the downstream side of the orifice was about 1/10 of the potential energy Δp which the air had in the fluidized bed.

DEVELOPMENT OF THEORETICAL EQUATIONS FOR THE PHENOMENA OF TWO-PHASE FLOW FROM THE ORIFICE

On the basis of the concepts illustrated above Equation (1) is changed to

$$\Delta p = \frac{\rho_A V^2_{AO}}{2\, g_c} + \frac{\rho_S V^2_{SO}}{2\, g_c} \frac{Q_S}{Q_A} + \Delta p_f \tag{2}$$

in which a pressure drop Δp_f, due to the filtration of the air through the solid, is considered together with the terms accounting for the kinetic energies of the gas and solid flowing out of the orifice. Equation (2) is an energy balance referred to the unit volume of gas outflowing.

In addition an equation of the type

$$V_{SO} = f(V_{AO}) \tag{3}$$

must be written, which accounts for the transport of the solid.

Two hypotheses are made in deriving Equations (2) and (3): the effect of the net downward velocity of the particles on the outflow conditions is negligible, and the friction forces between the particles on the upstream side of the orifice may be disregarded.

Of these assumptions the first seems likely because when the ratio of the diameter of the column to the diameter of the orifice is relatively high, the vertical velocities of the particles are small compared with their efflux horizontal velocity. In the case of the experiments carried out in this study the ratio of the vertical to the horizontal particle velocity ranged from 1/900 to about 1/150.

On the contrary the second assumption brings noticeable limitations into the Equations (2) and (3). Later the possibility that such limitations may be responsible for some discrepancies between theoretical and experimental results will be considered.

Another assumption has been made in developing Equations (2) and (3). It assumes that the properties of the solid-gas system, upstream from the orifice, in a hemispherical shell which has as its center the center of the orifice, depend only on the distance of the shell from the center of the efflux orifice. A schematic diagram appears in Figure 5 which is based on this hypothesis. It shows that at a radius r from the center O the pressure and the fraction voids are constant. Under such conditions the flow rate, the velocities of the solid, and gas are given by $Q_A/2\pi r^2\epsilon$ and $Q_S/2\pi r^2(1-\epsilon)$ respectively. The relative velocity between the air and the solid (slip velocity) in the direction of the radius of the hemisphere is

$$V_R = \frac{1}{2\pi r^2}\left(\frac{Q_A}{\epsilon} - \frac{Q_S}{1-\epsilon}\right)$$

At points sufficiently far from the orifice the average value of V_R becomes very small because of the fact that both V_A and V_S decrease inversely as the square of the radius r. At such points the system is supposed to be fully fluidized and composed of particles moving in disorderly motion with an average velocity of zero. These particles are held in suspension by the air which has an average velocity equal to that which is fluidizing the bed. As the distance from the orifice is progressively reduced, the value of V_R increases and reaches a maximum value at the orifice.

The hemispherical symmetry of the velocity of the gas and solid makes the analytical treatment of the problem easier. It should be pointed out however that this hypothesis under certain aspects oversimplifies the efflux process. For instance it clearly cannot be valid in the space between the efflux section and the hemisphere of radius r_o (Figure 5). In fact in this space the flux section of the solid and of the gas is halved passing from $2\pi r_o^2$ to πr_o^2.

Thus in developing Equations (2) and (3) the values of the gas and solid velocities and of the slip velocities given before will be changed as follows:

$$V_A = \frac{Q_A}{\pi r^2 \epsilon};\; V_S = \frac{Q_S}{\pi r^2(1-\epsilon)};$$

$$V_R = \frac{1}{\pi r^2}\left(\frac{Q_A}{\epsilon} - \frac{Q_S}{1-\epsilon}\right) \tag{4}$$

This is equivalent to assuming a spherical symmetry restrained to the cone indicated by dotted lines in Figure 5.

With this concept in mind the two phenomena, the flow of the air filtering through the solid and the transport of solid, will be examined separately.

This analysis aims at determining an analytical expression for Δp_f in Equation (2) and making explicit Equation (3).

Filtration of the air

The pressure drop occurring in the current of air as it flows through an element of a hemisphere of thickness dr can be determined by modifying the Carman-Kozeny correlation (5) as follows:

$$(\rho_A V_R)^{1.9} = \frac{\rho_A\, \epsilon^{1.1}\, g_c\, D_p^{1.1}\, \Phi^{1.1}}{2.9\,(1-\epsilon)^{1.1}\, \mu_A^{0.1}} \frac{dp}{dr} = K_1 \frac{dp}{dr} \tag{5}$$

Such a differential equation can be easily integrated when one assumes that the fraction of voids remains constant over the field of integration taken under consideration. This field of integration is considered to consist of a very small volume of suspension which is situated immediately upstream from the orifice.

Integrating Equation (5) for a constant ϵ and using Equation (4) to determine V_R one obtains

$$\left[\frac{Q_A}{\epsilon} - \frac{Q_S}{1-\epsilon}\right]^{1.9} \int_{r_o}^{R_o} \frac{dr}{r^{3.8}} = \left(\frac{\pi}{\rho_A}\right)^{1.9} K_1 \int_{p(r_o)}^{p(R_o)} dp \tag{6}$$

The condition of the fluidized bed at this radius is characterized by a pressure and fraction of voids equal to those which exist in the column at the same average height.

The equation of the outflow is therefore

$$\left[\frac{Q_A}{\epsilon} - \frac{Q_S}{1-\epsilon}\right]^{1.9} \left(\frac{1}{2.8}\right) \left[\frac{1}{r_o^{2.8}} - \frac{1}{R_o^{2.8}}\right] = \left(\frac{\pi}{\rho_A}\right)^{1.9} K_1\, \Delta p_f \tag{7}$$

This equation may be simplified by considering that the term $1/R_o^{2.8}$ is very small when compared with $1/r_o^{2.8}$. The result is

$$\left[\frac{Q_A}{\epsilon} - \frac{Q_S}{1-\epsilon}\right]^{1.9} \frac{1}{2.8\, r_o^{2.8}} = \left(\frac{\pi}{\rho_A}\right)^{1.9} K_1\, \Delta p_f \tag{8}$$

Drag of the Solid

Consideration is given first to an element of volume dw contained in the two hemispherical shells which have radii r and $r + dr$. The quantity of solid has a mass $\rho_p(1-\epsilon)\, dw$.

If the frictional resistance is excluded, the particles contained in the differential volume dw are subjected to two kinds of forces: dynamical forces caused by the different velocities with which the gas moves with respect to the solid, and inertial forces which constitute the reaction of the solid to the thrust generated by the dynamical forces. Equating these two forces and supposing a stationary flow one obtains

$$\frac{d\,V_S}{dt}\, \rho_p(1-\epsilon)dw =$$

$$-\frac{d V_S}{dr} V_S \rho_p (1-\epsilon) dw = \rho_A \frac{C'_D (V_A - V_S)^2}{2} \frac{(1-\epsilon) dw\, s_p}{w_p} \quad (9)$$

The negative sign before the second term indicates that the velocity V_S increases progressively as the particles approach the orifice, that is as the value of r decreases.

From Equation (9) one gets

$$-\frac{d V_S}{dr} V_S = \frac{C'_D \rho_A s_p}{2\, w_p\, \rho_p} (V_A - V_S)^2 = K_2 (V_A - V_S)^2 \quad (10)$$

Equation (10) can be changed to

$$-\frac{d (V_S)^2}{2\, dr} = K_2 (V_A - V_S)^2 \quad (11)$$

and integration of Equation (11) gives

$$-\int_{r_o}^{R_o} d(V_S)^2 = \int_{r_o}^{R_o} 2K_2 (V_A - V_S)^2\, dr \quad (12)$$

When one employs Equation (4) to get an expression for V_R and assumes constant values of ϵ and C'_D, contained in K_2, integration gives

$$[-V_S^2]_{r_o}^{R_o} = -\frac{2K_2}{\pi^2 \epsilon^2} \left(Q_A - Q_S \frac{\epsilon}{1-\epsilon} \right)^2 \left[\frac{1}{3} \frac{1}{r^3} \right]_{r_o}^{R_o} \quad (13)$$

Equation (13) may be simplified when one considers that the term $1/R_o^3$ is very small compared with $1/r_o^3$. Thus

$$\frac{Q_S^2}{\pi^2 (1-\epsilon)^2 r_o^4} = \left(Q_A - Q_S \frac{\epsilon}{1-\epsilon} \right)^2 \frac{1}{3} \frac{1}{r_o^3} \frac{2K_2}{\pi^2 \epsilon^2} \quad (14)$$

and upon simple rearrangement the result is

$$\frac{Q_A}{Q_S} = \frac{\epsilon}{1-\epsilon} \left(1 + \sqrt{\frac{3}{2K_2\, r_o}} \right) \quad (15)$$

Special consideration is required in the selection of the drag coefficient which occurs in the expression of K_2 in Equation (10). To begin one could employ charts which give such values for a single particle as a function of the Reynolds number (6), provided they are properly modified. It would be useful for example to take into account the influence exerted on the drag coefficient by the presence of the other particles, similarly to the formula proposed by Ladenburg for the case of the motion of a sphere of diameter D_p in a cylinder of diameter L (7):

$$C'_D = C_D \left[1 + 2.1 \frac{D_p}{L} \right]$$

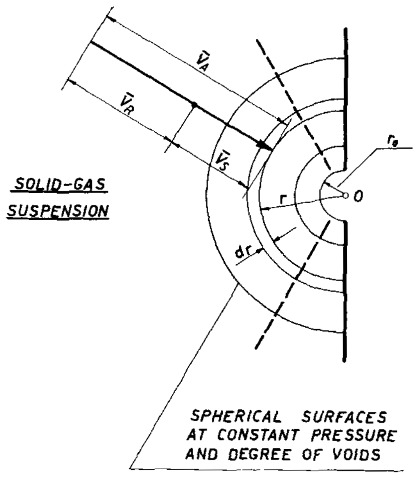

Fig. 5. Diagram of analytical model.

With such a correction the drag coefficients relative to the motion of a single sphere C_D are transformed into those related to the motion of a group of particles characterized by a fraction of voids ϵ:

$$C'_D = C_D [1 + 2.1(1-\epsilon)] \quad (16)$$

Equation (2), with Δp_f given by Equation (8), and Equation (15) represent the analytical expression for the phenomena which influence the process of the flow of a solid-gas suspension from an orifice. These equations permit a determination of the flow rates of solid and gas flowing through an orifice and out of a fluidized solid-gas system as a function of the radius or diameter of the orifice, of the characteristics of the solid particles (density and average diameter) and of the air (density and viscosity), and of the pressure drop which is determined by the passage of the air through the fluidized bed.

COMPARISON BETWEEN EXPERIMENTAL RESULTS AND THE THEORETICAL EQUATIONS

A comparison between the actual results and those predicted by Equations (2) and (15) is now in order. This comparison could be made if the values of the flow of solid and gas, the pressure drop, the radius of the orifice, and the fraction voids of the system in the region of the orifice are all known. Such a verification cannot be carried out directly because the value of the fraction voids is unknown at the throat of the orifice.

Nevertheless the validity of the analytical model can be verified indirectly.

At first Equation (2) is simplified by disregarding the terms $\frac{\rho_A V^2_{AO}}{2\, g_c} + \frac{\rho_p V^2_{SO}}{2\, g_c} \frac{Q_S}{Q_A}$ which, as considered above, are small compared with Δp. Then Δp is substituted for Δp_f in Equation (8) which is arranged as follows:

$$\left[\frac{Q_A}{\epsilon} - \frac{Q_S}{1-\epsilon} \right] \frac{1}{\Delta_p^{0.52}} = \sqrt[1.9]{\frac{2.8\, \pi^{1.9} \epsilon^{1.1} g_c \Phi^{1.1} D_p^{1.1}}{2.9\, \rho_A^{0.9} (1-\epsilon)^{1.1} \mu_A^{0.1}}}\, r_o^{1.47} \quad (17)$$

In accordance with this Equation the experimental data obtained in the group of runs carried out with the same materials should fall on a straight line passing through the origin and have a slope of

$$K_1 = \sqrt[1.9]{\frac{2.8\, \pi^{1.9} \epsilon_m^{1.1} g_c D_p^{1.1} \Phi^{1.1}}{2.9\, \rho_A^{0.9} (1-\epsilon_m)^{1.1} \mu_A^{0.1}}}$$

in a plot with coordinates of $\left[\frac{Q_A}{\epsilon_m} - \frac{Q_S}{1-\epsilon_m} \right] \frac{1}{\Delta p^{0.52}}$ and $r_o^{1.47}$. For the same values of ϵ_m the ratio Q_A/Q_S must satisfy the relation of Equation (15) in the diagram of Q_A/Q_S and r_o. Such a verification was carried out for all the materials used in the experiments. For each series of runs the average fraction voids, which for the various flow conditions (pressure and diameter of the orifice) satisfy Equation (17), was determined by trial, and then the theoretical curves which represent Equation (15) were plotted and compared with the experimental data.

In general one can affirm that the agreement between the theoretical equation and the experimental results is satisfactory, particularly if account is taken of the simplifications adopted in the formulation of the equations and in the integration of the differential Equations (5) and (9). For every material there is an average value of the degree of voids ϵ_m, which value roughly satisfies Equation (17) and gives a fair agreement between the experimental values of Q_A/Q_S and the theoretical values obtained from Equation (15).

Calculated values of ϵ_m range between 0.45 and 0.50 for all the materials, except sawdust, for which a higher value of ϵ_m was found. This fact suggested correlating the experimental data with the assumption in Equations (15) and (17) of a value of $\epsilon_m = 0.50$ for every material. The correlation was further simplified by considering that when the value of μ_A is small, as in the case of experiments in which air or other gas are used as fluidizing stream, Equation (17) may be modified by dropping $\mu_A^{0.1}$ and substituting exponents 0.5, 2, 1, and 1.5 for 0.52, 1.9, 0.9, 1.1, and 1.47. Equations (17) and (15) then become

$$\frac{Q_A - Q_S}{\left(\frac{g_c\, \Phi\, D_p\, \Delta p}{\rho_A} \right)^{0.5}} = 1.54\, r_o^{1.5} \quad (18)$$

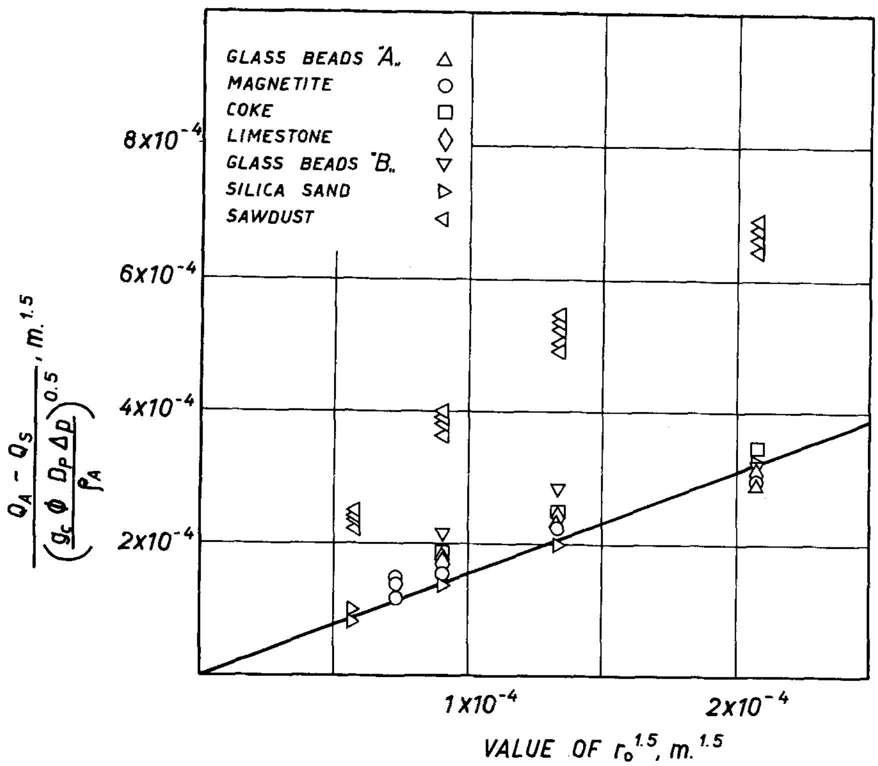

Fig. 6. Comparison between experimental and theoretical data obtained on the basis of the outflow equation which accounts for the phenomenon of the filtration of air through the solid particles suspension.

$$\frac{Q_A}{Q_S} = 1 + \sqrt{\frac{3}{2 K_2 r_o}} \qquad (19)$$

Figures 6, 7, and 8 show the verification of Equations (18) and (19).

Some reasons may be suggested to explain the deviations between the experimental data and the theoretical results.

First it must be considered that, as the port diameter becomes larger, the simplifications introduced in the integration of the differential Equations (5) and (9), which are based on the assumption that the resistance to the filtration of the air is concentrated immediately upstream from the orifice, become more and more unacceptable.

Then a difference between the real conditions of flow and those predicted by the theoretical model must be pointed out. In accordance with the model the hemispheres shown in Figure 5 are characterized by a void fraction which is constant with time and which depends only on the nature and size of the particles and on the mean outflow pressures. In reality the fluctuations in pressure which one observes at the bottom of the fluidized bed disturb the efflux and in particular the stability of the above said hemispheres which are continuously destroyed by the bubbles of gas and solid which move tumultuously through the bed and reformed by the air which flows through the orifice.

All this gives place to continuous oscillations of the actual quantity of gas and solid as well as a disparity between the average effective flow rates and those predicted on the basis of the efflux equations; the flow rates of air are greater than the theoretical and the flow rates of the solid are less. Therefore in order to verify Equations (18) and (19) with experimental data one should use different values of the degree of voids depending on the intensity of fluctuations, which in turn depends on the nature and diameter of the solid particles and on the ratio between the height and the diameter of the fluidized bed. Considering that different efflux pressures were obtained in the tests using different heights of bed one has to take a secondary effect of the pressure into account. Presumably the fluctuations of the pressure at the bottom of the bed have contributed to give the deviations of Figures 6 and 8 for the data obtained in experiments with sawdust. These experiments were the only ones done without the screens, and therefore it is possible that they have been characterized by perturbations in the steady conditions of the pressures at the base of the bed which are frequent and intense. However an additional influence of a possible agglomeration of the sawdust on the above said deviations cannot be excluded.

The effect of the frictional forces, which were not taken into account in the formation of the differential equations, must also be considered. Such forces increase as the ratio between the diameter of the particle and the diameter of the orifice grows and cause more air to leave than would be expected, Equation (19). A comparison between the theoretical values and

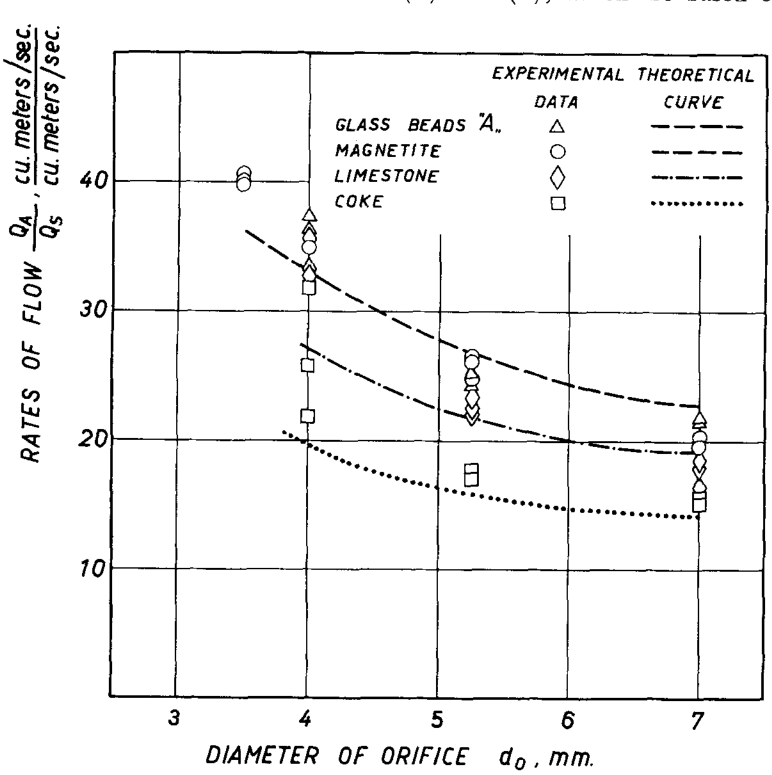

Fig. 7. Comparison between experimental and theoretical data obtained on the basis of the outflow equation which accounts for the phenomenon of the drag of the solid particles on the part of the effluxing air.

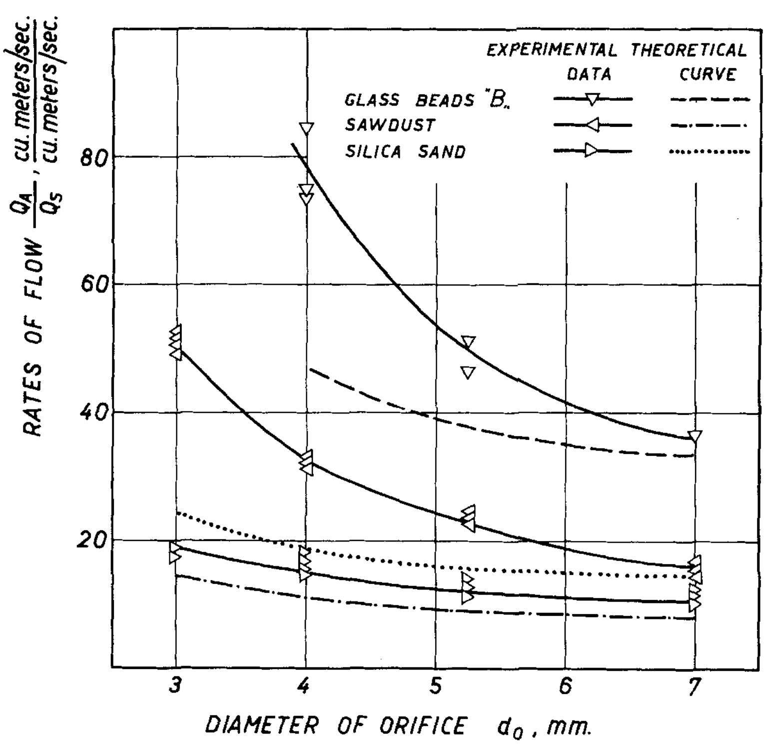

Fig. 8. Comparison between experimental and theoretical data obtained on the basis of the outflow equation which accounts for the phenomenon of the drag of the solid particles on the part of the effluxing air.

the experimental values in the tests with silica sand and glass beads shows this clearly (Figure 8).

CONCLUSIONS

From these tests it appears easy to produce jets of solid pulverized material by making orifices in the walls of the fluidized column.

In consequence of the difference in pressure between the interior and exterior of the column the air flows from the orifices and drags with it solid particles which comprise the fluidized bed.

The experiments show that for a given material the flow rate of the leaving solid depends solely upon the diameter of the orifice and on the efflux pressure.

A slight influence is exercised on the efflux flow rate by the flow rate of the fluidizing air at least in the range of the port diameters investigated.

The results of the tests show also that the ratio between the flow rate of air to that of the solid is practically independent of the outflow pressure. For a given material these ratios depend only on the diameter of the orifice and become progressively reduced with the increase in the diameter.

The analytical model proposed is useful in considering the two phenomena of the filtration of the air and of the drag of the solid from which is determined the efflux. The two differential equations found on the basis of the analytical model are verified in major part by the experimental data obtained. However the simplifications involved in the analytical model and the approximations used in the integration of the differential equations are of a different degree in experiments with the same materials with different port diameters and different efflux pressures. Also the simplifications differ in degree under the same experimental conditions but when different materials are used. The nature of the effect of these simplifications must be pointed out to make precise the limit to which the diagrams of Figures 6 through 8 can be extended to predict the volume flow rate of solids and gas from a fluidized bed under conditions of test different from those used in this study.

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NOTATION

C_D, C'_D = drag coefficients, dimensionless
d_o = diameter of the outflow port, m.
D_p = mean particle diameter, m.
g_c = conversion factor, 9.8 kg. m./kg. (force) sec.^{2}
K_1, K_2 = dimensional constants
L = cylinder diameters in the Ladenburg correction factor, m.
Δp = outflow pressure, mm. water
Δp_f = pressure drop for the filtration of the air through the particle suspension, mm. water
Q_A = outflow air flow rate, cu. m.
Q_S = outflow solid flow rate, cu. m.
r = radius of the hemisphere in the outflow model, m.
R_o = radius of the external hemisphere in the outflow model, m.
r_o = radius of the outflow port, m.
s_p = surface of the particle exposed to the air stream, sq. m.
t = time, sec.
V_A = air velocity, m./sec.
V_S = solid velocity, m./sec.
V_{A_o} = air velocity at the outflow port, m./sec.
V_{S_o} = solid velocity at the outflow port, m./sec.
V_R = slip velocity between air and solid, m./sec.
V_F = velocity of a continuous medium, m./sec.
w_p = volume of the particle, cu. m.

Greek Letters

ϵ = void degree, dimensionless
ϵ_m = mean void degree which satisfies Equation (17), dimensionless
μ_A = air viscosity, kg./(m.)(sec.)
ξ = efflux coefficient, dimensionless
ρ_A = air density, kg./(cu. m.)
ρ_p = solid density, kg./(cu. m.)
ρ_F = density of a continuous medium, kg./(cu. m.)
Φ = shape factor for the Carman-Kozeny correlation, dimensionless

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